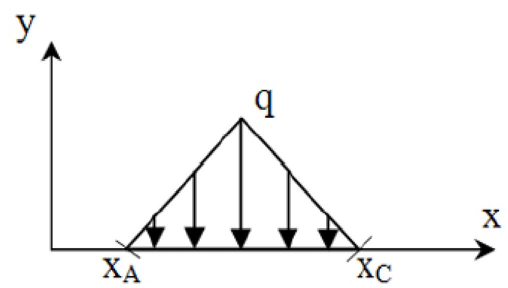
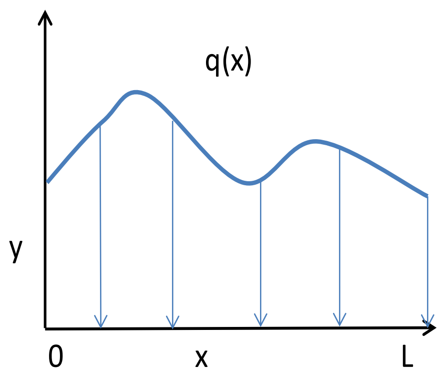
Find the resultant force and moment and the support straight line of the next distributed load, considering x_a=2 m, x_q=4 m, x_c=6 m

Note: q is in N/m



Solution:

When a generic continuous distribution of load like the next one is given:



It is possible to calculate the resultant force and moment it creates just by integrating the curve, following the next process:

$$\mathrm{F} = \int_0^L \mathrm{q(x).dx}$$

$$\mathrm{M} = \int_0^L \mathrm{q(x).x.dx}$$

And then for the calculation of the support straight line:

$$M = AP \ x \ F$$

But AP and F are perpendiculars, so:

$$AP = \frac{M}{F}$$

Once reviewed these points, we begin to solve the exercise. The first step is to determine q(x). In this case this task can be easily done using the next expression:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Thus from x_a to x_q:

q(x=2) = 0

q(x=4) = q

$$\frac{q(x) - 0}{q - 0} = \frac{x - 2}{4 - 2} \rightarrow q(x) = \frac{q}{2}(x - 2)$$

And from x_q to x_c

q(x=4) = q

q(x=6) = 0

$$\frac{q(x) - q}{0 - q} = \frac{x - 4}{6 - 4} \rightarrow q(x) = \frac{q}{2}(6 - x)$$

Once q(x) is obtained we will proceed with the calculation of the resultant force:

$$\mathrm{F} = \int_{x_a}^{x_c} \mathrm{q(x).dx} = \int_2^4 \frac{q}{2}(x - 2).\mathrm{dx} +$$

$$+ \int_4^6 \frac{q}{2}(6 - x).dx = \left[\frac{q}{2}\frac{(x-2)^2}{2}\right]_2^4 +$$

$$+ \left[\frac{q}{2}\frac{(6-x)^2}{2}\right]_6^4 = \mathrm{q + q = 2q}$$

Then: $\mathbf{F = 2q}$ **(N)**

This could have been also quickly calculated by simply considering that the area below of the curve is:

$$A = 2.\frac{1}{2}q.L = 2.\frac{1}{2}q.2 = 2q$$

Now for the calculation of the resultant moment:

$$M = \int_{x_a}^{x_c} q(x).x.dx = \int_{2}^{4} \frac{q}{2}(x-2).x.dx +$$

$$+\int_{4}^{6} \frac{q}{2}(6-x).x.dx = \left[\frac{q}{2}\left(\frac{x^3}{3}-x^2\right)\right]_2^4 +$$

$$+\left[\frac{q}{2}\left(\frac{x^3}{3}-3x^2\right)\right]_6^4 = \frac{q}{2}\left(\frac{4^3}{3}-4^2\right) -$$

$$-\frac{q}{2}\left(\frac{2^3}{3}-2^2\right) + \frac{q}{2}\left(\frac{4^3}{3}-3.4^2\right) -$$

$$\frac{q}{2}\left(\frac{6^3}{3}-3.6^2\right) = \frac{q}{6}(16+4-80+108)$$

$$\mathbf{M = 8q\ (N.m)}$$

Finally for calculating the support straight line:

$$\mathbf{AP = \frac{8q\ N.m}{2q.N} = 4\ m}$$

Then, there is an equivalent system to the first one, just made up a simple punctual load of 2q at a distance of 4 meters from the origin.