In the structure ABCDEFG shown in the next figure, C is a pin, E and G are roller supports, and A is a simply support.

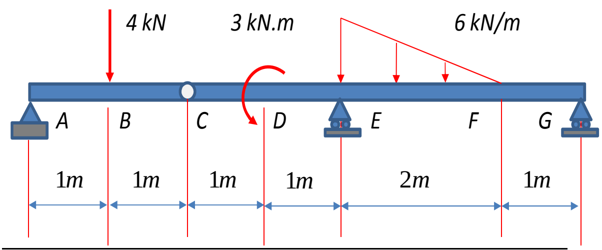
Determine:

a) Reaction forces (**H_A, V_A, V_E, V_G**) using the equations of equilibrium.

b) Shear forces and Bending moment laws in the beam. Analytical expression as a function of x (horizontal coordinate from A).

c) Draw the Shear force and Bending moment diagrams of the beam (showing maximum values).

d) Knowing that the maximum allowable normal stress of the cross section of the beam is 100 MPa and considering a safety coefficient equal to 4, determine the HEB profile that optimizes the structure.



Solution:

a) Reaction forces

$$\sum M_C = 0 \rightarrow 2V_A - 4 = 0 \rightarrow \mathbf{V_A = 2\ kN}$$

$$\sum M_G = 0$$

$$2.7 - 4.6 - 3 + 3V_E - \frac{1}{2}.6.2.\left(1 + \frac{4}{3}\right) = 0$$

$$\mathbf{V_E = 9\ kN}$$

$$\sum F_y = 0 \rightarrow \mathbf{V_G = -1\ kN}$$

$$\sum F_x = 0 \rightarrow H_A = 0 \rightarrow \mathbf{H_A = 0\ kN}$$

b) Force laws

$0 \leq x \leq 1$

$$2 - V_1 = 0 \rightarrow \mathbf{V_1 = 2\ kN}$$

$$2x - M_1 = 0 \rightarrow \mathbf{M_1 = 2x\ kN.m}$$

$1 \leq x \leq 3$

$$V_2 = 2 - 4 \rightarrow \mathbf{V_2 = -2\ kN}$$

$$M_2 = 2x - 4(x - 1) \rightarrow \mathbf{M_2 = -2x + 4\ kN.m}$$

$3 \leq x \leq 4$

$$\mathbf{V_3 = -2\ kN}$$

$$\mathbf{M_3 = -2x + 1\ kN.m}$$

$4 \leq x \leq 6$

$$q(x) = q_0\left(1 - \frac{x}{l}\right) = 6\left(1 - \frac{x}{2}\right)$$

$$V_4 = -2 + 9 - V_T$$

$$V_T(x) = \int_0^{x-4} 6\left(1 - \frac{\mathfrak{z}}{2}\right) d\mathfrak{z} = \left[6\mathfrak{z}\left(1 - \frac{\mathfrak{z}}{4}\right)\right]_0^{x-4}$$

$$V_T(x) = 6(x - 4)\left(1 - \frac{x - 4}{4}\right)$$

$$\mathbf{V_4 = 7 - 6(x - 4)\left(1 - \frac{x - 4}{4}\right)}$$

$$M_4 = -2x + 1 + 9(x - 4) - M_T$$

$$M_T(x) = \int_0^{x-4} 6\mathfrak{z}\left(1 - \frac{\mathfrak{z}}{4}\right) d\mathfrak{z} = \left[6\left(\frac{\mathfrak{z}^2}{2} - \frac{\mathfrak{z}^3}{12}\right)\right]_0^{x-4}$$

$$M_T(x) = 6(x - 4)^2\left(\frac{1}{2} - \frac{x - 4}{12}\right)$$

$$M_4 = 7x - 35 - 6(x - 4)^2\left(\frac{1}{2} - \frac{x - 4}{12}\right)$$

$$M_{max} \rightarrow \frac{dM(x)}{dx} = V(x) = 0 \rightarrow \nexists x \in (4,6)$$

$$M_{max} = M_4(x = 4) = -7\ kN.m$$

$$6 \leq x \leq 7$$

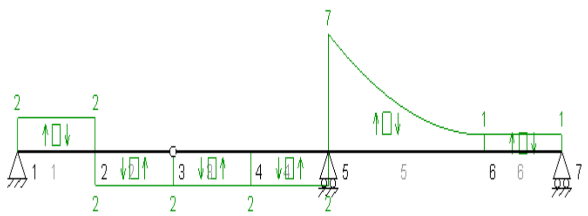
$$\mathbf{V_5 = 1\ kN}$$

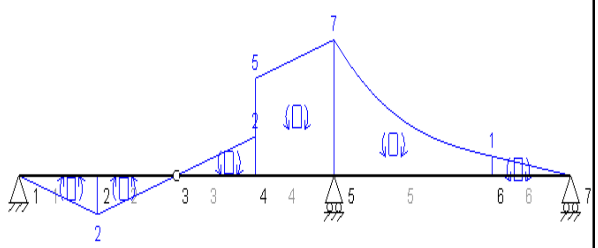
$$M_5 = z\ kN.m$$

Considering that z is the horizontal coordinate starting from G (x +z = 7):

$$\mathbf{M_5 = 7 - x\ kN.m}$$

c) <u>Force laws diagrams</u>





d) <u>Selection of the HEB-profile</u>

$$\sigma_{max} = \frac{M_{max}.y_{max}}{I_x} = \frac{M_{max}}{\left(\frac{I_x}{y_{max}}\right)} = \frac{M_x}{W_x}$$

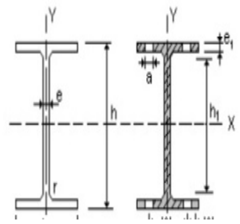
$$\sigma_{all} = \frac{100\ MPa}{4} = 25\ MPa$$

$$W_x \geq \frac{M_{max}}{\sigma_{all}}$$

$$W_x \geq \frac{7\ kN.m}{25\ MPa} = 280\ cm^3$$

And now entering the chart:

Perfiles HEB



Perfil	Dimensiones							Términ			
	h mm	b mm	e mm	e_1 mm	r mm	h_1 mm	u mm	A cm^2	S_x cm^3	I_x cm^4	W_x m^3
HEB 100	100	100	6,0	10	12	56	567	26,0	52,1	450	90
HEB 120	120	120	6,5	11	12	74	686	34,0	82,6	864	144
HEB 140	140	140	7,0	12	12	92	805	43,0	123	1509	216
HEB 160	160	160	8,0	13	15	104	918	54,3	177	2492	311
HEB 180	180	180	8,5	14	15	122	1040	65,3	241	3831	426

$$216 < 280\ cm^3 < 311 cm^3 \rightarrow \mathbf{HEB - 160}$$