MECÁNICA DE ESTRUCTURAS 23-Enero -2019 [redacted]

APELLIDOS………………………………………………………………………… NOMBRE……………………………………………

Grupo…………. Titulación……

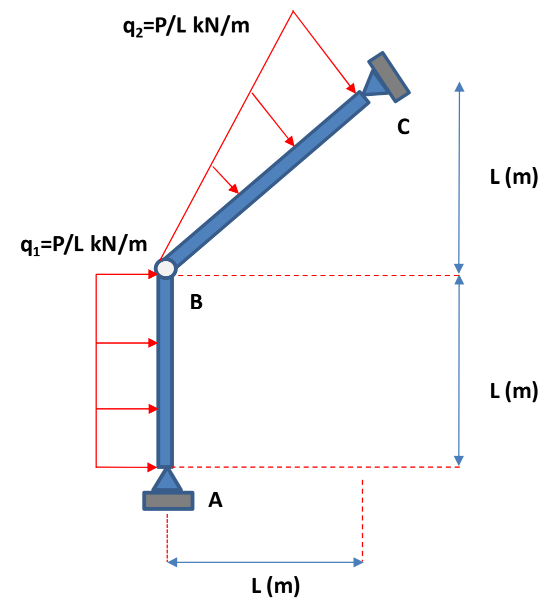
PROBLEMA 1. (TIEMPO MÁXIMO 50 MIN.)

En la estructura **ABC** de la Figura 1, **A** y **C** son apoyos simples y **B** es una rótula. La barra **AB** está sometida a una carga distribuida uniforme de valor **q_1=P/L** kN/m y la barra **BC** está sometida a una carga distribuida de tipo triangular de valor máximo **q_2=P/L** kN/m. Se pide:

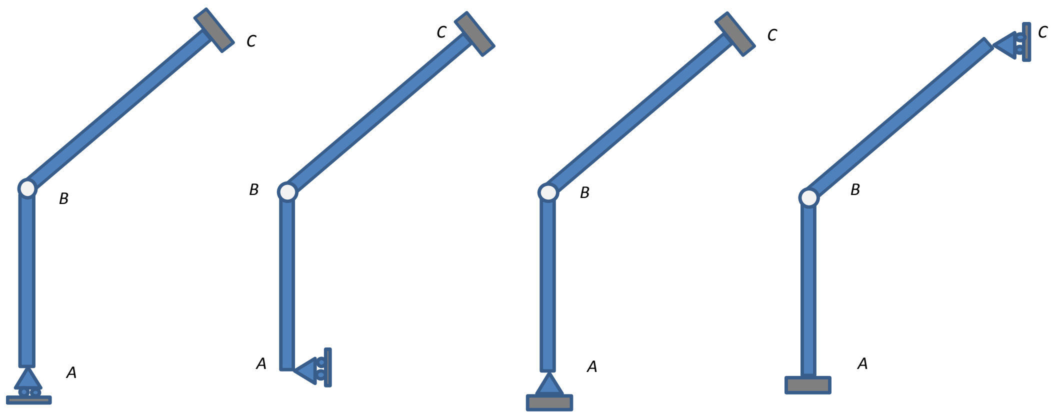
a) Determinar las reacciones en los apoyos en función de **P** y **L**. (**2 puntos**).

b) Tomando **L = 1** m y **P = 1** kN determinar las leyes analíticas de esfuerzo cortante y momento flector en toda la estructura, dibujando sus correspondientes diagramas. (**4 puntos**).

c) Utilizando exclusivamente el diagrama de esfuerzos cortantes de la estructura obtenido en **b)**, verificar dónde se obtiene el máximo y el mínimo momento flector en la barra **AB**. (**1 punto**).



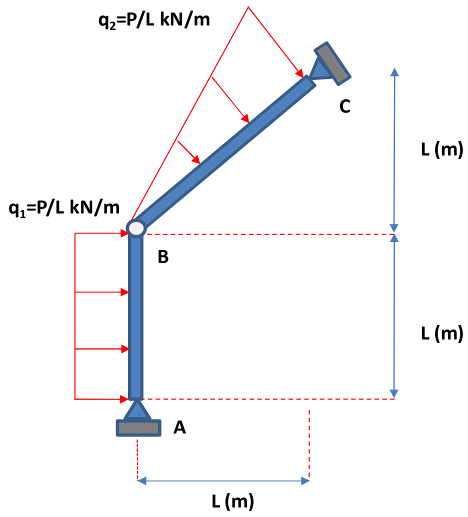
d) Considere ahora la estructura anterior, con el mismo sistema de cargas pero con diferente configuración de apoyos como se muestra en las figuras (A), (B), (C) y (D). Evalúe cada opción desde un punto de vista estructural y establezca cuál de ellas experimenta el menor valor de momento flector en el tramo **AB**. (**3 puntos**).



(A) (B) (C) (D)

Nota: Para la calificación del problema se considerará la claridad y justificación de las respuestas.

EXERCISE 1



1) Reaction forces calculation.

For this calculation, the continuous distributed triangle force has been reduced to its equivalent punctual forces in the vertical and horizontal axes. This is $F_x = F_y = P/2$ kN at two thirds of its total length from B.

$$\sum M_B^{AB} = 0 \rightarrow H_A.L - \frac{P.L}{L}\frac{L}{2} = 0 \rightarrow \mathbf{H_A} = \frac{\mathbf{P}}{\mathbf{2}}\ \mathbf{kN}\ (\leftarrow)$$

$$\sum F_x = 0 \rightarrow H_A + H_C = \frac{3P}{2} \rightarrow \mathbf{H_C} = \mathbf{P}\ \ \mathbf{kN}\ (\leftarrow)$$

$$\sum M_B^{BC} = 0 \rightarrow V_C.L + P.L - 2.\frac{2L}{3}\frac{P}{2} = 0 \rightarrow \mathbf{V_C} = \frac{\mathbf{P}}{\mathbf{3}}\ \mathbf{kN}\ (\downarrow)$$

$$\sum F_y = 0 \rightarrow V_A - \frac{P}{3} = \frac{P}{2} \rightarrow \mathbf{V_A} = \frac{\mathbf{5P}}{\mathbf{6}}\ \mathbf{kN}\ (\uparrow)$$

2) Force laws diagrams (P = L =1)

Stretch AB

$$\mathbf{V_{AB}(x)} = \frac{\mathbf{1}}{\mathbf{2}} - \mathbf{x}$$

$$\mathbf{M_{AB}\ (x)} = \frac{\mathbf{x}}{\mathbf{2}} - \frac{\mathbf{x^2}}{\mathbf{2}}$$

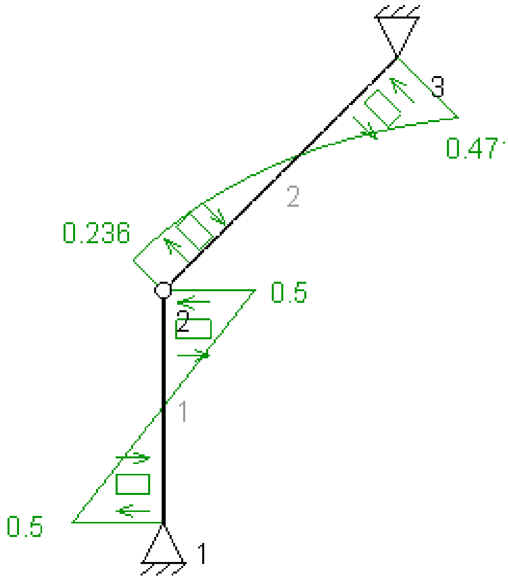
$$M_{max} \rightarrow \frac{dM(x)}{dx} = V(x) = 0 \rightarrow x = 0.5\ m \rightarrow M_{max} = 0.125\ kN.m$$

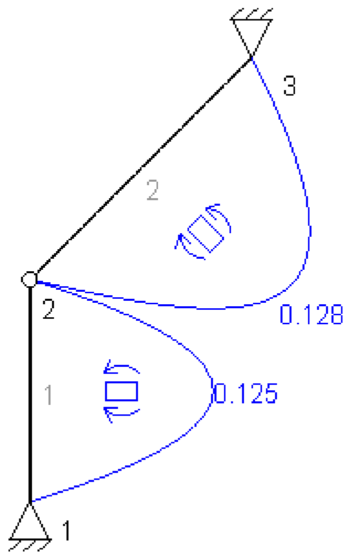
Stretch BC

$$q(x) = \frac{x}{\sqrt{2}} \rightarrow V_T(x) = \int_0^x q(\mathfrak{z})d\mathfrak{z} = \int_0^x \frac{\mathfrak{z}}{\sqrt{2}}d\mathfrak{z} = \frac{x^2}{2\sqrt{2}} \rightarrow \mathbf{V_{BC}(x)} = \frac{\sqrt{\mathbf{2}}}{\mathbf{6}} - \frac{\mathbf{x^2}}{\mathbf{2\sqrt{2}}}$$

$$V_T(x) = \frac{x^2}{2\sqrt{2}} \rightarrow M_T(x) = \int_0^x V(\mathfrak{z})d\mathfrak{z} = \int_0^x \frac{\mathfrak{z}^2}{2\sqrt{2}}d\mathfrak{z} = \frac{x^3}{6\sqrt{2}} \rightarrow \mathbf{M_{BC}(x)} = \frac{\sqrt{2}\mathbf{x}}{\mathbf{6}} - \frac{\mathbf{x^3}}{\mathbf{6\sqrt{2}}}$$

$$M_{max} \rightarrow \frac{dM(x)}{dx} = V(x) = 0 \rightarrow x = 0.816\text{ m} \rightarrow M_{max} = M_{BC}(x = 0.816) = 0.128\text{ kN.m}$$



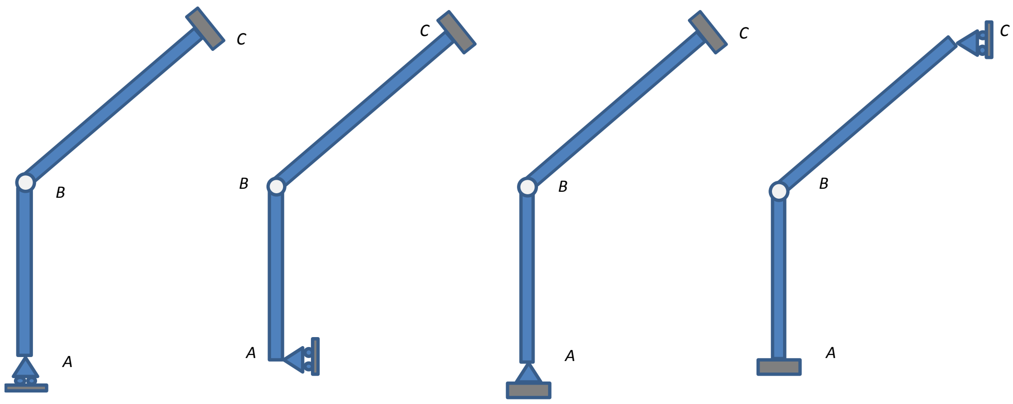


3) <u>Verify the values of the bending.</u>

$$M(x = 0.5) - M(x = 0) = \frac{1}{2}0.5^2 \rightarrow \mathbf{M(x = 0.5) = 0.125\ kN.m}$$

$$M(x = 1) - M(x = 0) = \frac{1}{2}0.5^2 - \frac{1}{2}0.5^2 \rightarrow \mathbf{M(x = 1) = 0\ kN.m}$$

4) <u>Structure assessment and minimum bending at AB.</u>



(A) (B) (C) (D)

a) DSI = 0, but the structure is a mechanism.

b) DSI = 0, statically determined. The maximum bending at AB can be easily determined by means of the expression below:

$$M_{max} = \frac{qL^2}{8} = 0.125\text{ kN.m}$$

c) DSI = 1, then it is statically indeterminate, and it cannot be calculated using exclusively the equilibrium equations.

d) DSI = 0, statically determined:

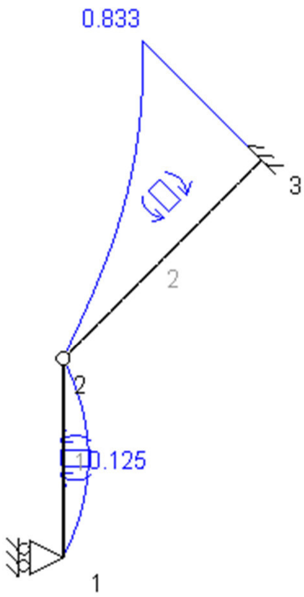
$$\sum M_B^{BC} = 0 \rightarrow H_c L - 2.\frac{2L}{3}\frac{1}{2} = 0 \rightarrow \; H_C = \frac{2}{3}\ \mathrm{kN}\ (\leftarrow)$$

$$\sum F_x = 0 \rightarrow \; H_A + H_C = \frac{3}{2} \rightarrow \; H_A = \frac{5}{6}\ \mathrm{kN}\ (\leftarrow)$$

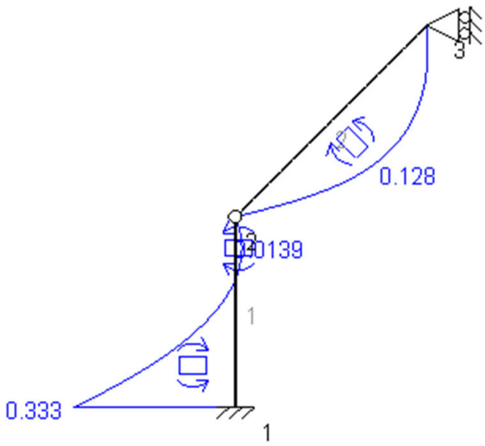
$$\sum M_B^{AB} = 0 \rightarrow M_A - \frac{5L}{6} + \frac{L}{2} = 0 \rightarrow M_A = \frac{1}{3}\ \mathrm{kN.m}$$

$$M_A = \frac{1}{3}\ \mathrm{kN.m} \rightarrow M(x = 0) = 0.333\ \mathrm{kN.m} > 0.125\ \mathrm{kN.m}$$

Therefore, the structure with the lowest bending is option b)



Structure b)



Structure d)