EXERCISE: in the structure ABCDEFG shown in the figure, A is a roller support, C a joint, and G is a clamped support. Determine:

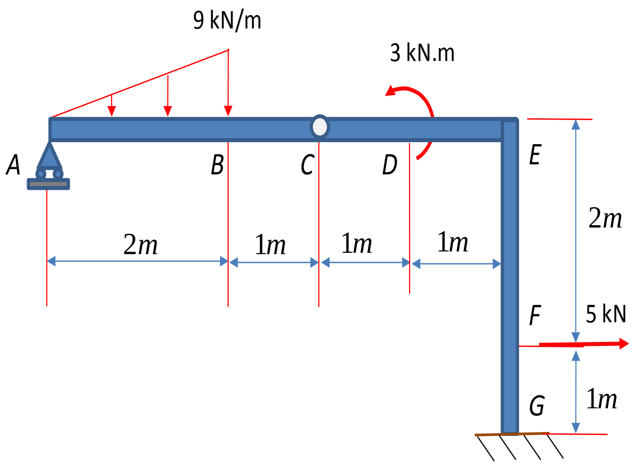
a) Internal, external and global degree of static indeterminacy. Explain the implications of these results

b) Reaction forces (**V**V_A, H_G, V_G, M_G) using the equations of equilibrium.

c) Calculate the reaction force at support A (V_A) in the structure of the exercise by the principle of virtual work.

d) Normal and Shear force and Bending moment laws. Analytical expressions and diagrams.

e) Normal and Shear force and Bending moment diagrams.



a) **Stability**

$$EDSI = 4 - 3 = 1$$

$$IDOF = 3(3-1) = 6$$

$$IL = 2.(2-1) + 2.(3-1) = 5$$

$$IDSI = 5 - 6 = -1$$

$$DSI = EDSI + IDSI = 1 - 1 = 0$$

Therefore, the structure is completely linked and the reactions are statically determined as it is going to be shown in the next chapter.

b) **Reaction forces calculation**

$$\sum M_c = 0 \rightarrow 3V_A - \frac{1}{2}.9.2.\left(1+\frac{2}{3}\right) = 0 \rightarrow$$

$$\mathbf{V_A = 5\ kN}$$

$$\sum F_y = 0 \rightarrow \mathbf{V_G} = \frac{1}{2}.9.2 - 5 = \mathbf{4\ kN}$$

$$\sum F_x = 0 \rightarrow \ \mathbf{H_G = -5\ kN}$$

$$\sum M_G = 0$$

$$5.5 - 9.\left(3+\frac{2}{3}\right) - 3 + 5.1 + M_G = 0$$

$\mathbf{M_G = 6\ kN.m}$ (clockwise)

c) **PVW**

$$\sum \delta W_{virt} = \sum F\delta v_i + M\delta\,\varphi_i = 0$$

$$V_A.\delta v_1 - 9.\delta v_T = 0$$

$$\delta v_1 = 3.\delta\varphi$$

$$\delta v_T = \left(1+\frac{2}{3}\right)\delta\varphi$$

$$V_A.3\delta\varphi - 9.\frac{5}{3}\delta\varphi = 0$$

$$\mathbf{V_A = 5\ kN}$$

Verifying the previous result.

d) **Force laws**

$$0 \le x \le 2$$

$$q(x) = q_0\frac{x}{L} = 9\frac{x}{2} = 4{,}5x$$

$$V_T(x) = \int_0^x 4{,}5.\,z.\,dz = \frac{9}{4}x^2$$

$$V_{AB} = 5 - \frac{9}{4}x^2$$

$$M_T(x) = \int_0^x \frac{9}{4}.\,z^2.\,dz = \frac{9}{4}\left[\frac{z^3}{3}\right]_0^x = \frac{3}{4}x^3$$

$$M_{AB} = 5x - \frac{3}{4}x^3$$

$$M_{max} \rightarrow \frac{dM(x)}{dx} = V(x) = 0 \rightarrow x = \sqrt{\frac{20}{9}} = 1{,}49 \text{ m}$$

$$M_{AB}\ (x = 1{,}49) = 4{,}97 \text{ kN.m}$$

$$2 \leq x \leq 4$$

$$V_{BD} = 5 - 9 = -4 \text{ kN}$$

$$M_{BD} = 5x - 9\left(x - \frac{4}{3}\right) = -4x + 12 \ \text{ kN.m}$$

$$4 \leq x \leq 5$$

$$V_{DE} = -4 \text{ kN}$$

$$M_{BD} = -4x + 9 \ \text{ kN.m}$$

$$0 \leq y \leq 1$$

$$N_{FG} = -4 \text{ kN}$$

$$V_{FG} = 5 \text{ kN}$$

$$M_{FG} = 5y + 6$$

$$1 \leq y \leq 3$$

$$N_{FE} = -4 \text{ kN}$$

$$V_{FE} = 5 - 5 = 0 \text{ kN}$$

$$M_{FG} = 5y + 6 - 5(y - 1) = 11 \text{ kN.m}$$

Now with all the information and inputs calculated, it is possible to draw the force laws diagrams:

e) Force Laws diagrams