Exercise 1: *The equation of a cable subjected to its own weight (catenary) and supported at two points located at the same height (thus, the cable is symmetric), is $y = 200\,cosh(0{,}02.x)$. Tension in a point where the tangent forms 60° with the horizontal is 4 kN. Determine:*

a) Minimum tension (T_0) and weight per unit of length (p).

b) Consider now a distance of 240 m between the supports. Calculate the maximum tension at the cable and its length.

Solution:

a) Minimum tension

$y = a\cosh(x/a) = 200\cosh 0{,}02x$

then $a = 200\,m$.

$T_o = T\cos\alpha = 4.\cos 60° \rightarrow \boldsymbol{T_o = 2\,kN}$

$a = 200\,m = \frac{T_o}{p} = \frac{2\,kN}{p} \rightarrow \boldsymbol{p = 10\,N/m}$

b) Tension and length

We work with half of the cable, so x is:

$x = \frac{L}{2} = \frac{240}{2} = 120\,m$

$y = a\cosh\left(\frac{x}{a}\right) = 200\cosh\left(\frac{120}{200}\right) \cong 237\,m$

$T_{max} = p.y_{max} = 2370\,N$

$T_y = \sqrt{T_{max}^2 - T_o^2} \cong 1271{,}57\,N$

$s = \sqrt{y^2 - a^2} = 127{,}33\,m$

$\boldsymbol{s_{tot} = 2.s = 254{,}66\,m}$

Verification of the value of s and T_y

$s = a.\sinh\left(\frac{x}{a}\right) = 200.\sinh\left(\frac{120}{200}\right) = 127{,}3\,m$

$\boldsymbol{s_{tot} = 2.s = 254{,}66\,m}$

$T_y = \frac{p.s_{tot}}{2} = \frac{10\,{}^{N}/_{m}.254{,}6\ m}{2} = 1271{,}57\,N$

As it is seen, both methods lead to the same result.

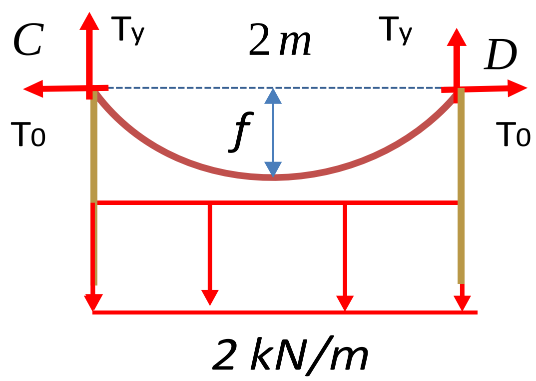
Exercise 2: *A cable is subjected to a uniform continuous load (parabolic cable) of 2 kN/m and its deflection at the middle-span is 0,1 m.*

a) Determine the tension at the supports C and D knowing the distance between them is 2 m.

b) Vectorial expression of the tension at C and D.

Solution:

$D_{CD} = 2\,m;\ f = 0{,}1\,m; q = 2\,kN/m$



a) Tension

$y - y_o = f = 0{,}1 = \frac{q}{2T_o}x^2 = \frac{2\frac{kN}{m}}{2T_o}.\left(\frac{D_{CD}}{2}\right)^2$

$\boldsymbol{T_o = 10\,kN}$

$$T_y = \frac{q.L}{2} = \frac{2\frac{kN}{m}.2m}{2}$$

$$\boldsymbol{T_y = 2\ kN}$$

$$T_C = T_D = \sqrt{T_o^2 + T_y^2} = 10{,}19\ kN$$

b) Vectorial expressions.

$$\boldsymbol{T_C = (-10, 2)\ kN}$$

$$\boldsymbol{T_D = (10, 2)\ kN}$$

Exercise 3: *A cable following the equation y=(2/3).x*^{1,5} *is suspended between two points located at x=0 and x=1. Calculate its total length.*

Solution:

$$ds^2 = dy^2 + dx^2 \to ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$L = \int_0^1 \sqrt{1 + (x^{0,5})^2}dx = \left[\frac{(1+x)^{3/2}}{3/2}\right]_0^1 =$$

$$= \frac{4\sqrt{2}}{3} - \frac{2}{3} \to \boldsymbol{L = \frac{2}{3}(2\sqrt{2} - 1)\ m}$$