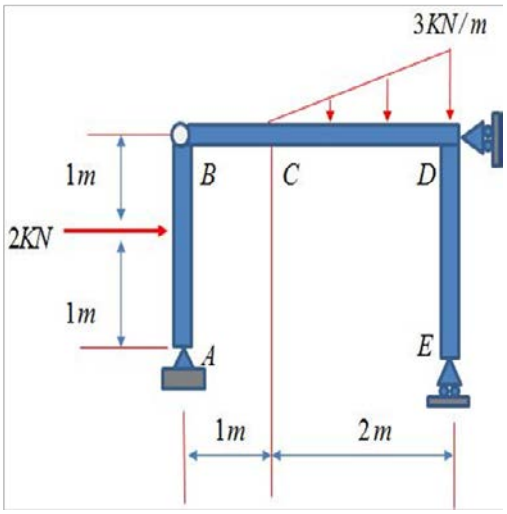
For the structure given in the figure, determine:

a) Degree of static indeterminacy and its implications.

b) Reaction forces at the supports.

c) Force laws in the whole structure (normal and shear forces as well as and bending).

d) Force laws diagrams obtained in part d) of the exercise.



a) Stability

$EDSI = 4 - 3 = 1$

$IDOF = 3(2 - 1) = 3$

$IL = 2(2 - 1) = 2$

$IDSI = 2 - 3 = -1$

$DSI = EDSI + IDSI = 1 - 1 = 0$

The structure is entirely linked and the reactions exerted by the supports can be determined just by using the three scalar equations of equilibrium.

b) Reaction calculation

$$\sum M_B = 2H_a - 2.1 = 0 \rightarrow \mathbf{H_a = 1\ kN}\ \text{(left)}$$

$$\sum F_x = 0 \rightarrow H_D = 2 - 1 \rightarrow \mathbf{H_e = 1\ kN}\ \text{(left)}$$

$$\sum M_e = 0 \rightarrow 3V_a + 2.1 - \frac{1}{2}.2.3.\frac{2}{3} - 2.1 = 0$$

$$\mathbf{V_a = {}^{2}/_{3}\ kN}$$

$$\sum F_y = 0 \rightarrow V_e = \frac{1}{2}.3.2 - \frac{2}{3}$$

$$\mathbf{V_e = {}^{7}/_{3}\ kN}$$

c) Force laws

$\{(0 \le y \le 1) \cap (x = 0)\}$

$$\mathbf{N_1 = -\frac{2}{3}\ kN}$$

$$\mathbf{V_1 = 1\ kN}$$

$$\mathbf{M_1 = y\ kN.m}$$

$\{(1 \le y \le 2) \cap (x = 0)\}$

$$\mathbf{N_2 = -\frac{2}{3}\ kN}$$

$$V_2 = 1 - 2 \rightarrow \mathbf{V_2 = -1\ kN}$$

$$M_2 = y - 2(y - 1)$$

$$\mathbf{M_2 = -y + 2\ kN.m}$$

$\{(0 \le x \le 1) \cap (y = 2)\}$

$$\mathbf{N_3 = -1\ kN}$$

$$\mathbf{V_3 = \frac{2}{3}\ kN}$$

$$M_3 = \frac{2}{3}x - 2.1 + 1.2 \rightarrow \mathbf{M_3 = \frac{2}{3}x\ kN.m}$$

$\{(1 \leq x \leq 3) \cap (y = 2)\}$

$$q(x) = \frac{qx}{L} = \frac{3x}{2}\ \frac{kN}{m}$$

$$V_T(y) = \int_0^{x-1} \frac{3\xi}{2} d\xi = \left[\frac{3\xi^2}{4}\right]_0^{x-1}$$

$$V_T = \frac{3}{4}(x-1)^2$$

$$M_T(y) = \int_0^{x-1} \frac{3\xi^2}{4} d\xi = \left[\frac{\xi^3}{4}\right]_0^{x-1}$$

$$M_T = \frac{(x-1)^3}{4}$$

$$\mathbf{V_4 = \frac{2}{3} - \frac{3}{4}(x-1)^2\ kN}$$

$$\mathbf{M_4 = \frac{2x}{3} - \frac{(x-1)^3}{4}\ kN.m}$$

$$M_{max} \rightarrow \frac{dM(x)}{dx} = V(x)$$

$$V(x) = -0{,}75x^2 + 1{,}5x - 0{,}083\ kN = 0$$

$$x = 1{,}93\ m$$

$$\mathbf{M_{max} = M_4(x = 1{,}93) = 1{,}09\ kN.m}$$

$\{(x = 3) \cap (0 \leq y \leq 2)\}$

$$\mathbf{N_5 = -\frac{7}{3}kN}$$

$$\mathbf{V_5 = 0\ kN}$$

$$\mathbf{M_5 = 0\ kN.m}$$

d) Force laws diagrams

At this point there is enough information to completely define the force laws and bending moment diagrams that are shown in the next page: